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## Abstract

We propose a two-phase model having a Newtonian rheology for the fluid phase and friction for the particle phase to describe bed-load transport in laminar pipe flows. This model is able to provide a description of the flow of the mobile granular layer. When the flow is not significantly perturbed, simple analytical results of the particle flux varying cubically with the Shields number can be obtained. This prediction compares favorably with experimental observations of the time-evolution of the bed height in conditions of laminar flow. A simple linear stability analysis based on this particle flux also accounts reasonably well for the dune formation observed.

## Introduction

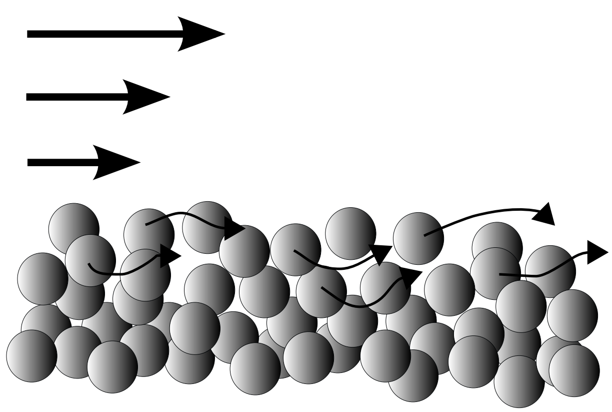

**Figure 1:** Bed-load transport.

When particle beds are submitted to shearing flows, the particles at the surface of the bed can move as soon as hydrodynamic forces acting on them exceed a fraction of their apparent weight. Bed-load refers to the sediment in transport that is carried by intermittent contact with the stream-bed by rolling, sliding, and bouncing, as shown in Fig. 1. This situation occurs in a wide variety of natural phenomena, such as sediment transport in rivers or by air, and in industrial processes, such as hydrate or sand issues in oil production and granular transport in food or pharmaceutical industries. A very common feature that arises is the formation of ripples, i.e. small waves on the bed surface, or of dunes, i.e. larger mounds or ridges.

The widely recognized mechanism for dune or ripple formation is the fluid inertia or more precisely the phase-lag between the bottom shear stress and the bed waviness generated by the fluid inertia, see e.g. Charru & Hinch (2006) and references therein. In that case, the shear stress, the maxima of which are slightly shifted upstream of the crests, drags the particles from the troughs up to the crests. However, a complete description of the bed instability is still lacking as the coupling between the granular media and the fluid is poorly understood. Empirical and/or phenomenological laws relating the particle flux to the bottom shear stress have been used in the literature.

The situation addressed here is that in which the bed-load can be considered as a mobile granular medium where the particles mainly interact through contact forces. In the past decade, advances has been made in the understanding of granular flows. In particular, it has been shown that a simple rheological description in terms of a friction coefficient may be sufficient to capture the major properties of granular flows (GDR Midi 2004). This description has been found to be also successful when an interstitial fluid is present (Cassar, Nicolas & Pouliquen 2005). We have chosen to use this rheology to describe the granular phase.

We have used a two-phase model having a Newtonian rheology for the fluid phase and friction for the particulate phase. The model equations have been solved numerically in one-dimension and analytically in asymptotic cases. This continuum approach is able to provide an onset of motion for the particle phase and a description of the flow of the mobile granular layer. At some distance from threshold, we obtain the simpler analytical result of the particle flux varying cubically with the Shields number. This algebraic formulation seems quite satisfactory for describing experimental observations of bed-load transport in pipe flows (Ouriemi *et al.* 2009 Part 1). Based on this particle flux, a simple linear stability analysis has been performed to predict the threshold for dune formation. This basic analysis accounts reasonably well for the experimental observations for dune formation (Ouriemi *et al.* 2009 Part 2).

## The two-phase model

We have chosen to use the standard two-phase equations, see e.g. Jackson (2000), and to propose some closures appropriate to the present problem.

The equations of continuity for the fluid and the particle phases are respectively

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial(\epsilon u_i^f)}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial(\phi u_i^p)}{\partial x_i} = 0, \tag{2}$$

where $u_i^f$ is the local mean fluid velocity, $u_i^p$ the local mean particle velocity, $\phi$ the particle volume fraction, and $\epsilon = 1 - \phi$ the void fraction or fraction of space occupied by the fluid.

The momentum equations for the fluid and particle phases are respectively

$$\rho_f \frac{D_f(\epsilon u_i^f)}{Dt} = \frac{\partial \sigma_{ij}^f}{\partial x_j} - n f_i + \epsilon \rho_f g_i, \tag{3}$$

$$\rho_p \frac{D_p(\phi u_i^p)}{Dt} = \frac{\partial \sigma_{ij}^p}{\partial x_j} + n f_i + \phi \rho_p g_i, \tag{4}$$

where $g_i$ is the specific gravity force vector, $\rho_f$ the fluid density, $\rho_p$ the particle density, $n$ the number density (number of particles per unit volume). The force $f_i$ represents the average value of the resultant force exerted by the fluid on a particle. The stress tensors $\sigma_{ij}^f$ and $\sigma_{ij}^p$ may be regarded as effective stress tensors associated with the fluid and particle phases, respectively. We need some closure for the interphase force and stresses of the two phases.

The interphase force can be decomposed into a generalized buoyancy force and a force which gathers all the

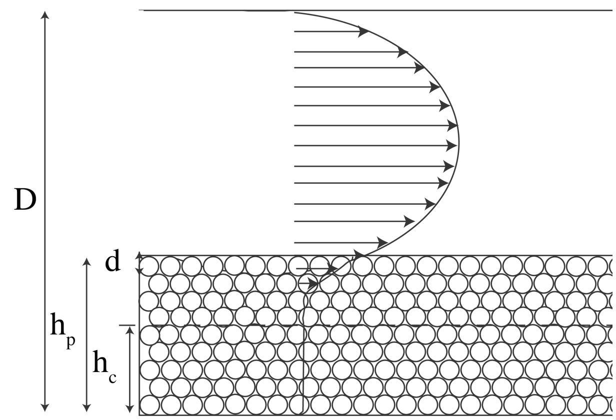


**Figure 2:** Sketch of a particle bed submitted to a Poiseuille flow in a two dimensional channel.

remaining contributions (Jackson 2000). In the present bed-load case, the remaining contribution reduces to the dominant viscous Darcy drag. The effective stress tensor associated with the fluid phase is supposed to be of Newtonian form with an effective viscosity $\eta_e$ (that for simplicity we can take as the Einstein viscosity) while the stress tensor of the particle phase comes only from direct particle-particle interactions and is described by a Coulomb friction model where:

- the tangential stress is proportional to the load (i.e. the particle pressure $p^p$) when the granular shear rate is positive (i.e. is equal to $\mu p^p$ with a friction coefficient $\mu$ which mostly depends upon the particle geometry and which is given by the tangent of the angle of repose),

- the tangential stress is indeterminate when the granular shear rate is zero.

## Bed-load transport in pipe flow

In the calculation, we have considered a flat bed of thickness $h_p$ consisting of particles having a diameter $d$. This bed is submitted to a stationary and uniform Poiseuille flow in a two dimensional channel of thickness $D$, as depicted in Fig. 2. The two-phase equations presented above are shown to reduce to the momentum equation for the mixture (particles + fluid) and the Brinkman equation for the fluid velocity. Calculations of bed-load transport have been performed numerically but also analytically at some distance from threshold where the mobile layer is larger than a grain size. The details of these calculations can be found in Ouriemi *et al.* (2009 Part 1).

Here we present a much simpler calculation which gives the flavor of the physical mechanisms involved. The mo-

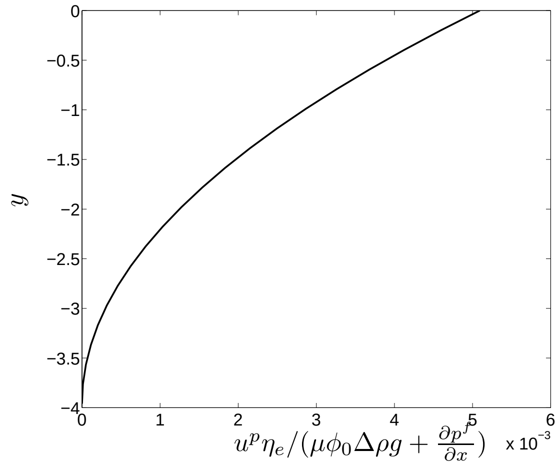


**Figure 3:** Typical parabolic profile predicted inside the bed.

mentum equation of the mixture can be written as

$$\tau^p(y) + \tau^f(y) = \tau^f(h_p) - \frac{\partial p^f}{\partial x}(h_p - y), \qquad (5)$$

where $\partial p^f/\partial x$ is the constant fluid pressure gradient driving the Poiseuille flow. This equation shows that the mixture shear stress $\tau^f + \tau^p$ increases linearly with depth from the surface value due to the horizontal pressure gradient $\partial p^f/\partial x$. It also describes the exchange between the shear stress of the fluid phase $\tau^f$ and that of the solid phase $\tau^p$. On the one hand, at the top of the granular bed $h_p$, the particle shear stress is zero and increases inside the bed until it reaches $\mu p^p$ where the granular medium starts to be sheared. The particle pressure is proportional to the apparent weight of the solid phase

$$p^p = \phi_0 \Delta\rho g (h_p - y), \qquad (6)$$

and increases inside the bed. The particle shear stress can keep the value $\mu p^p$ until it reaches $y = h_c$ inside the bed. On the other hand, at the top of the granular bed $h_p$, the fluid shear stress is equal to $\tau^f(h_p)$ and goes to zero at $h_c$ below which the granular medium is immobile and behaves as a porous medium. Moreover, the Darcy drag term is dominant in the Brinkman equation for the fluid velocity. Therefore, inside the bed, there is very little slip between the two phases and both particle and fluid phases move at the velocity of the mixture with the following parabolic profile, see Fig. 3,

$$u^p \approx u^f \approx \frac{(\mu\phi_0\Delta\rho g + \frac{\partial p^f}{\partial x})}{\eta_e}\frac{(y - h_c)^2}{2}. \qquad (7)$$

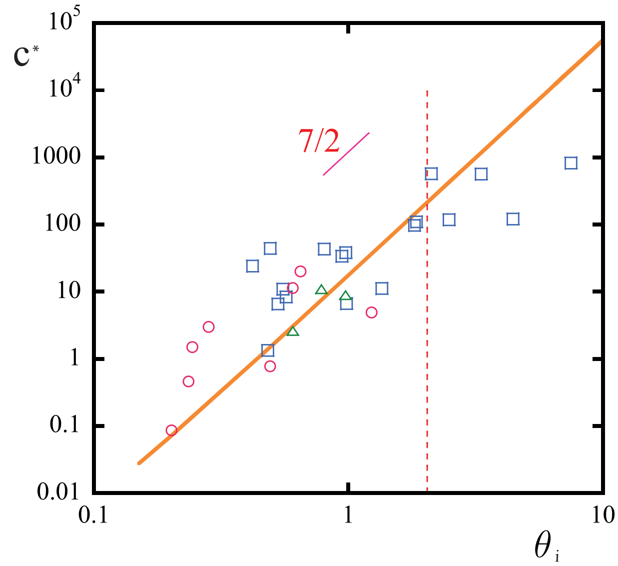


**Figure 4:** Dimensionless kinematic velocity as a function of the initial Shields number for different combinations of particles and fluids. The solid line shows the prediction $c^*(\theta) \propto \frac{\partial q_p}{\partial \theta} \propto \theta^{7/2}$. The vertical dotted line shows the limit of this prediction for large $\theta$.

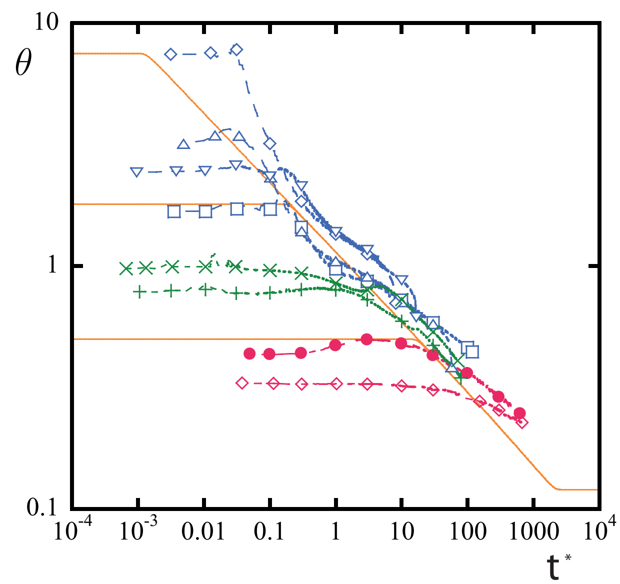


**Figure 5:** Temporal evolution of the bed height for different combinations of particles and fluids. The solid lines represent the numerical integration of equation (11). The time-scale is $(6Re/Ga)^{1/2}(D/d)(\eta_e/\Delta\rho g d)$ where $Re$ is the Reynolds number and $Ga$ the Galileo number $Ga = \rho_f \Delta\rho g d^3/\eta^2$.

The bed-load thickness has a very simple linear variation in Shield number $\theta = \tau^f(h_p)/\Delta\rho g d$ (ratio of the shear stress at the top of the bed to the apparent weight of a single particle)

$$h_p - h_c \approx \frac{\theta d}{\mu\phi_0 + \frac{\partial p^f}{\partial x}/\Delta\rho g}. \quad (8)$$

Assuming that the critical Shields number for the onset of grain motion corresponds to the thickness of the mobile layer being half a particle, one has

$$\theta^c \approx \mu\phi_0/2. \quad (9)$$

Finally, the particle flux can be expressed as

$$q_p/\frac{\Delta\rho g d^3}{\eta_e} = \phi_0\frac{\theta^c}{24}\left(\frac{\theta}{\theta^c}\right)^3, \quad (10)$$

where $\eta_e$ is the effective viscosity of the fluid phase.

These predictions have been tested against experimental measurements of bed profile evolution in a pipe flow. The principle of the experiment is (i) to fill the pipe with fluid and particles and to build an uniform flat bed, and (ii) to record the evolution of the bed height as a function of time using a laser sheet illumination for a given flow rate. If we choose to be in the condition of bed-load transport, i.e. above the critical Shields number, the bed height is always seen to decrease with increasing time as the test section is not fed in with particles. The indirect method for measuring the flux of particles is to use the equation for the conservation of particles

$$\phi_0\frac{\partial h_p}{\partial t} + \frac{dq_p}{dh_p}\frac{\partial h_p}{\partial x} = 0, \quad (11)$$

which can be written as a kinematic wave equation in dimensionless form as a function of the nondimensional Shields number, see details in Ouriemi *et al.* (2009 Part 1).

There is good agreement between the experimental observations and the theoretical prediction based on the flux of particles found earlier in the new two-phase model. First, the prediction $\theta^c = \mu\phi_0/2$ with $\phi_0 = 0.55$ in the bulk of the bed and $\mu = 0.43$ (Cassar, Nicolas & Pouliquen 2005) agrees well with the experimental value $0.12 \pm 0.03$ (Ouriemi *et al.* 2007). Second, the cubic law for the particle flux seems satisfactory for describing the velocity of the kinematic waves which are triggered at the entrance of the tube, see Fig. 4, as well as the time-evolution of the bed height in conditions of bed-load transport for pipe flows, see Fig. 5.

## Dune formation in pipe flow

Different dune patterns are observed as the flow rate is increased from the laminar to the turbulent regimes. Small dunes present small amplitudes and only exist in laminar flow, see Fig. 6. Vortex dunes are characterized by the existence of vortices at their front and are found either in laminar or turbulent flow, see Fig. 7. Sinuous dunes, showing a double periodicity, appear in turbulent flow, see Fig. 8.

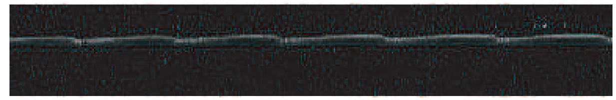

**Figure 6:** Profile of the small dunes.

**Figure 7:** Side and top views of the vortex dunes.

**Figure 8:** Top and side views of the sinuous dunes.

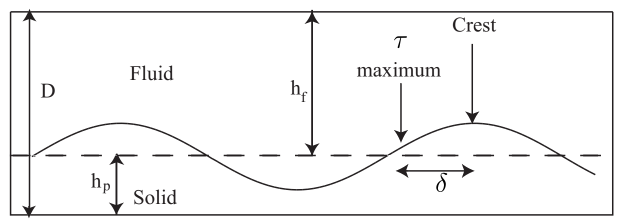


**Figure 9:** Destabilizing mechanism: The shear stress, the maxima of which are slightly shifted upstream of the crests, drags the particles from the troughs up to the crests

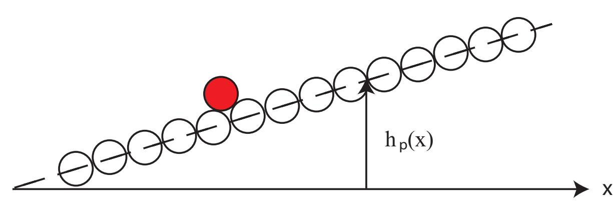


**Figure 10:** Stabilizing mechanism: Gravity force favors particle downhill motion

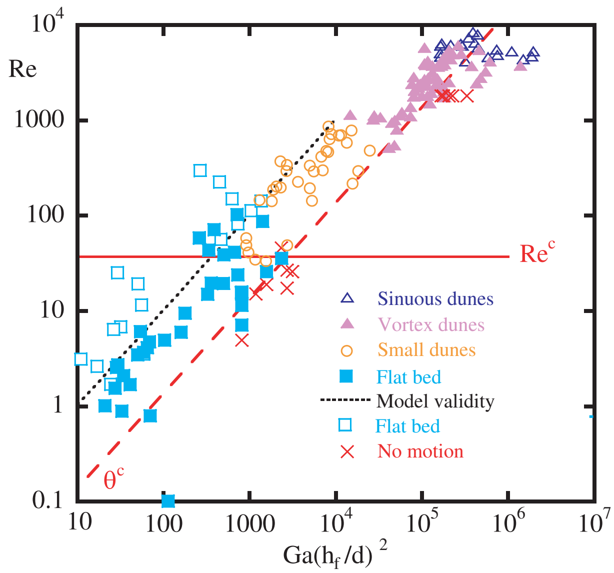


**Figure 11:** Phase diagram of the dune patterns.

To predict the small dune formation, we have performed a simple linear stability analysis where inertia in the fluid produces a phase-lag in the shear stress which is destabilizing, see Fig. 9, while the component of gravity down an incline stabilizes the perturbations, see Fig. 10. Following Charru & Hinch (2000), we first calculated the perturbed fluid flow over a wavy bottom considered as if fixed. Then, we used the particle flux found by Ouriemi *et al.* (2009 Part 1) to relate the bed height evolution to the shear stress at the top of the bed through the particle mass conservation. The threshold for dune formation is found to be controlled by the Reynolds number, see further details in Ouriemi *et al.* (2009 Part 2).

Figure 11 presents the phase diagram of the dune patterns in the plane $Re_{pipe}$, $Ga(h_f/d)^2$. We choose this plane to exhibit both the threshold for incipient particle motion controlled by the Shields number and that for dune instability predicted to be controlled by the Reynolds number in the linear stability analysis. In this plane, the predicted threshold for particle motion ($\theta^c = 0.12$) is the dashed line. The predicted instability threshold is the horizontal solid line. The dotted line indicates the domain of validity of the algebraic law relating the dimensionless particle flux to the Shields number found by Ouriemi *et al.* (2009 Part 1) and thus indicates the domain of validity of the instability threshold prediction of Ouriemi *et al.* (2009 Part 2).

The three regimes of ‘no motion’, ‘flat bed in motion’, and ‘small dunes’ are well delineated by these boundaries in the given limit of validity. Clearly, the threshold for incipient particle motion and that for small dune instability are observed to differ as there is a large region of ‘flat bed in motion’ without any dune formation. The threshold prediction of the simple linear stability with a single adjustable parameter that we have taken to be realistic, $Re_c \propto 1/\mu \approx 37.5$ with $\mu = 0.43$ (Cassar, Nicolas & Pouliquen 2005), provides a correct boundary for the ‘small dune’ instability. Furthermore, the regimes of ‘vortex dunes’ and ‘sinuous dunes’ seem separated and their thresholds also well described by the Reynolds number of the pipe as a control parameter.

## Conclusions

In summary, we have proposed a two-phase model to describe bed-load transport in the laminar viscous regime, i.e. the flux of particles in a flat mobile bed submitted to laminar flows. The fluid phase has been assumed to be a Newtonian viscous liquid with Einstein dilute viscosity formula applied to the concentrated situation while the particle phase to have Coulomb solid friction with the shear stress proportional to the pressure.

We have applied this continuum model to bed-load transport in pipe flows. The relevant equations have been found to be the Brinkman equation for the fluid phase and the momentum balance equation for the mixture. They have been solved numerically but also analytically. A very simple analytical model, valid when the Poiseuille flow is not significantly perturbed, finds that the bed-load thickness varies linearly with the Shields number whereas the particle flux cubically with it.

We have compared these predictions with experimental observations in pipe flow. The cubic law for the particle flux seems satisfactory for describing the velocity of the kinematic waves which are triggered at the entrance of the tube as well as the time-evolution of the bed height in conditions of bed-load transport for pipe flows.

We have also examined the different dune patterns which are observed when a bed composed of spherical particles is submitted to a shearing flow in a pipe. ‘Small dunes’ present small amplitudes and only exist in laminar flow. ‘Vortex dunes’ are characterized by the existence of vortices at their front and are found either in laminar or turbulent flow. ‘Sinuous dunes’, showing a double periodicity, appear in turbulent flow. While the threshold for incipient motion is determined by the Shields number, that for dune formation seems to be described by the Reynolds number.

To predict the small dune formation, we have performed a simple linear stability analysis. This analysis containing the basic ingredient of the destabilizing fluid inertia and stabilizing gravity is found sufficient to provide realistic predictions.

The two-phase nature of the problem has been only accounted for in the particle conservation equation in the present study and thus a full two-phase analysis should be performed in the future, in particular through full three-dimensional computations as undertaken in Chauchat & Médale (2010) and in the accompanying paper (Chauchat *et al.* 2010).

### Acknowledgement

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### References

Cassar, C., Nicolas & Pouliquen, O., Submarine granular flows down inclined planes, Phys. Fluids, Vol. 17, 103301, 2005

Chauchat J. and Médale M., A three-dimensional numerical model for incompressible two-phase flow of a granular bed submitted to a laminar shearing flow, Comput. Methods Appl. Mech. Engrg., Vol. 199, 439-449, 2010

Chauchat J., Ouriemi M., Aussillous P., Médale M., and Guazzelli É, A 3D Two-Phase Numerical Model for Sediment Transport, 7th International Conference on Multiphase Flow, ICMF 2010, Tampa, FL, May 30 – June 4, 2010

Charru, F. and Hinch, E. J. , 'Phase diagram' of interfacial instabilities in a two-layer Couette flow and mechanism of the long-wave instability, J. Fluid Mech., Vol. 414, 195-223, 2000.

Charru, F. and Hinch, E. J., Ripple formation on a particle bed sheared by a viscous liquid. Part 1. Steady flow, J. Fluid Mech., Vol. 550, pp 111-121, 2006.

GDR Midi, On dense granular flows, Eur. Phys. J., Vol. 14, pp 341-365, 2004.

Jackson, R., The dynamics of fluidized particles, Cambridge University Press, 2000

Ouriemi, M., Aussillous, P., Medale, M., Peysson, Y., and Guazzelli, E., Determination of the critical Shields number for particle erosion in laminar flow. Phys. Fluids, Vol. 19, 061706, 2007.

Ouriemi M., Aussillous P., and Guazzelli É., Sediment dynamics. Part 1: Bed-load transport by shearing flows, J. Fluid Mech., Vol. 636, pp 295-319, 2009

Ouriemi M., Aussillous P., and Guazzelli É., Sediment dynamics. Part 2: Dune formation in pipe flow, J. Fluid Mech., Vol. 636, pp 321-336, 2009